

佛光人文社會學院九十五學年度碩士班招生考試試題卷

系所別：資訊學系碩士班

科目：線性代數

用紙第 / 頁共 / 頁

1. Label the following statements as true or false. (每題 5 分)
- a) If matrix A is diagonalizable, that is $X^{-1}AX = D$, then the diagonalizing matrix X is unique.
 - b) Subsets of linearly dependent sets are linearly dependent.
 - c) Let $L: R^n \rightarrow R^n$ be a linear transformation. If $L(x_1) = L(x_2)$, then the vectors x_1 and x_2 must be equal.
 - d) If $a_1x_1 + a_2x_2 + \dots + a_nx_n = 0$ and $x_1, x_2, \dots, x_n$ are linearly independent, then all the scalars a_i are zero.
 - e) Every vector space that is generated by a finite set has a basis.
 - f) If $L: V \rightarrow W$ is a linear transformation, then $L(0_V) = 0_W$.
 - g) A vector space cannot have more than one basis.
 - h) If a vector space has a finite basis, then the number of vectors in every basis is the same.
 - i) If A is a 4×4 matrix of rank 1 and $\lambda = 0$ is an eigenvalue of multiplicity 3, then A is diagonalizable.
 - j) If A is a $n \times n$ matrix, then A and A^T have the same eigenvalues.
 - k) If V is a vector space having dimension n , then V has exactly one subspace with dimension 0 and exactly one subspace with dimension n .
 - l) Similar matrices have the same characteristic polynomials.
2. (20 分) Given $x_1 = [1, 1, 1]^T$ and $x_2 = [3, -1, 4]^T$:
- a) Do x_1 and x_2 span R^3 ? Explain.
 - b) Let x_3 be a third vector in R^3 and set $X = (x_1 \ x_2 \ x_3)$. What conditions would X have to satisfy in order for x_1, x_2, x_3 to form a basis for R^3 ?
 - c) Find a third vector x_3 that will extend the set $\{x_1 \ x_2\}$ to a basis for R^3 .
3. (20 分) Find the eigenvalues and the corresponding eigenspaces for the following matrices:

$$(a) A = \begin{bmatrix} -2 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} \quad (b) B = \begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix}$$