

佛光人文社會學院九十四學年度碩士班招生考試試題卷

系所別：資訊學系碩士班

科目：線性代數

用紙第 / 頁共 / 頁

1. Label the following statements as true or false. (每題 5 分)
 - a) If S is a linearly dependent set, then each vector in S is a linear combination of other vectors in S .
 - b) Any set containing the zero vector is linearly dependent.
 - c) Subsets of linearly dependent sets are linearly dependent.
 - d) Subsets of linearly independent sets are linearly independent.
 - e) If $a_1x_1 + a_2x_2 + \dots + a_nx_n = 0$ and $x_1, x_2, \dots, x_n$ are linearly independent, then all the scalars a_i are zero.
 - f) Every vector space that is generated by a finite set has a basis.
 - g) Every vector space has a finite basis.
 - h) A vector space cannot have more than one basis.
 - i) If a vector space has a finite basis, then the number of vectors in every basis is the same.
 - j) If S generates the vector space V , then every vector in V can be written as a linear combination of vectors in S in only one way.
 - k) Every subspace of a finite-dimensional space is finite-dimensional.
 - l) If V is a vector space having dimension n , then V has exactly one subspace with dimension 0 and exactly one subspace with dimension n .

2. Let R^+ denote the set of positive real numbers. Define the operation of scalar multiplication, denote $\circ$, by $\alpha \circ x = x^\alpha$ for each $x \in R^+$ and for any real number α . Define the operation of addition, denoted $\oplus$, by

$$x \oplus y = x \cdot y \text{ for all } x, y \in R^+$$

Thus for this system the scalar product of -3 times $\frac{1}{2}$ is given by

$$-3 \circ \frac{1}{2} = \left(\frac{1}{2}\right)^{-3} = 8$$

and the sum of 2 and 5 is given by $2 \oplus 5 = 2 \cdot 5 = 10$

Is R^+ a vector space with these operations? Prove your answer. (20 分)

$$3. \text{ Let } C = \begin{pmatrix} 1 & -1 & 0 & 1 \\ 1 & -1 & 0 & -1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & -1 & 0 \end{pmatrix}$$

- a) Calculate $\det(C)$ by cofactor expansion. (4 分)
- b) Are the row vectors of C linearly independent? (4 分)
- c) Are the column vectors of C linearly independent? (4 分)
- d) Find the inverse of C using elementary row operations on the augmented matrix $[C | I]$. (4 分)
- e) Find $\det(C \cdot C^T)$. (4 分)