

佛光人文社會學院九十三學年度碩士班招生考試試題卷

系所別：資訊學研究所

科目：資訊科學

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一.是非題

(每題兩分，共 30 分)

1. Label the following statements as true or false.

- If a vector space has a finite basis, then the number of vectors in every basis is the same.
- Two distinct eigenvectors corresponding to the same eigenvalue are always linearly dependent.
- Subsets of linearly independent sets are linearly independent.
- If S is a linearly dependent set, then each vector in S is a linear combination of other vectors in S .
- Any two eigenvectors are linearly independent.
- If S generates the vector space V , then every vector in V can be written as a linear combination of vectors in S in only one way.
- Every vector space that is generated by a finite set has a basis.
- The sum of two eigenvectors is always an eigenvector.
- Any set containing the zero vector is linearly dependent.
- Similar matrices always have the same eigenvalues.
- Every subspace of a finite-dimensional space is finite-dimensional.
- If V is a vector space having dimension n , then V has exactly one subspace with dimension 0 and exactly one subspace with dimension n .
- If two rows or columns of A are identical, then $\det(A) \neq 0$.
- If $a_1x_1 + a_2x_2 + \dots + a_nx_n = 0$ and $x_1, x_2, \dots, x_n$ are linearly independent, then all the scalars a_i are zero.
- If A, B are two $n \times n$ matrices, then $\det(AB) = \det(A) \times \det(B)$.

二.計算與程式題

(共 70 分)

1. Give an asymptotic upper and lower bound for the recurrence:(10 分)

$$T(n) = T\left(\left\lfloor \frac{2n}{10} \right\rfloor\right) + T\left(\left\lceil \frac{n}{10} \right\rceil\right) + \theta(n)$$

2. (a) Build a binary search tree for the following keys:(5 分)

12 89 4 61 22 10 52 91

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(b) If we assign the probabilities that each key being searched as follow:

Key k_i	12	89	4	61	22	10	52	91
Probability p_i	0.1	0.1	0.14	0.08	0.15	0.17	0.19	0.07

Find the structure of an optimal binary search tree with minimum search cost:

$$C = \sum_{i=1}^8 (\text{depth}(k_i) + 1) \cdot p_i, \text{ where } \text{depth}(k_i) \text{ is the depth of key } k_i \text{ in the optimal binary search tree. (15 分)}$$

3. Prove by induction that the i th Fibonacci number satisfies the equality

$$F_i = \frac{\phi^i - \hat{\phi}^i}{\sqrt{5}} \quad \text{where } \phi = \frac{1+\sqrt{5}}{2} \text{ is the golden ratio and } \hat{\phi} \text{ is its conjugate. (10 分)}$$

4. Consider a hash table of size $m=13$ and a corresponding hash function $h(k)=k \bmod m$. Illustrate the result of inserting keys, 22, 2, 31, 4, 15, 45, 28, 17, 88, 59, and 10 into the hash table using open addressing by linear probing. (10%)
5. Give an algorithm that takes a directed acyclic graph $G=(V, E)$ and two vertices s and t as input, and returns the number of paths from s to t in G . (10 分)
6. † An **Euler Tour** of a connected, directed graph $G=(V, E)$ is a cycle that traverses each edge of G exactly once, although it may visit a vertex more than once. Prove that a directed graph $G=(V, E)$ has an **Euler tour** if and only if $\text{in-degree}(v) = \text{out-degree}(v)$ for every $v \in V$. (10 分)

